

# An assessment of Evans' unified field theory II

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## Abstract

Evans developed a classical unified field theory of gravitation and electromagnetism on the background of a spacetime obeying a Riemann-Cartan geometry. In an accompanying paper I, we analyzed this theory and summarized it in nine equations. We now propose a variational principle for Evans' theory and show that it yields two field equations. The second field equation is algebraic in the torsion and we can resolve it with respect to the torsion. It turns out that for all physical cases the torsion vanishes and the first field equation, together with Evans' unified field theory, collapses to an ordinary Einstein equation.

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# 1 Introduction

In an accompanying paper [4], called I in future, we investigated the unified field theory of Evans [1, 2]. We take the notation and the conventions from I, where also more references to Evans' work can be found. We assume that the reader is familiar with the main content of part I before she or he turns her or his attention to the present paper. In I we were able to reduce Evans' theory to just nine equations, which we will list again for convenience.

Spacetime obeys in Evans' theory a Riemann-Cartan geometry (RC-geometry) that can be described by an orthonormal coframe  $\vartheta^\alpha$ , a metric  $g_{\alpha\beta} = \text{diag}(+1, -1, -1, -1)$ , and a Lorentz connection  $\Gamma^{\alpha\beta} = -\Gamma^{\beta\alpha}$ . In terms of these quantities, we can define torsion and curvature, respectively:

$$T^\alpha := D\vartheta^\alpha, \quad (1)$$

$$R_\alpha{}^\beta := d\Gamma_\alpha{}^\beta - \Gamma_\alpha{}^\gamma \wedge \Gamma_\gamma{}^\beta. \quad (2)$$

The Bianchi identities and their contractions follow therefrom.

Evans proposes an extended electromagnetic field with the potential  $\mathcal{A}^\alpha$ . By Evans' ansatz, this potential is postulated to be proportional to the coframe

$$\mathcal{A}^\alpha = a_0 \vartheta^\alpha, \quad (3)$$

with some constant  $a_0$ . The electromagnetic field strength is defined according to

$$\mathcal{F}^\alpha := D\mathcal{A}^\alpha. \quad (4)$$

The extended homogeneous and inhomogeneous Maxwell equations read in Lorentz covariant form

$$D\mathcal{F}^\alpha = R_\beta{}^\alpha \wedge \mathcal{A}^\beta \quad \text{and} \quad D^*\mathcal{F}^\alpha = {}^*R_\beta{}^\alpha \wedge \mathcal{A}^\beta, \quad (5)$$

respectively. Alternatively, with Lorentz non-covariant sources and with partial substitution of (3) and (4), they can be rewritten as

$$d\mathcal{F}^\alpha = \Omega_0 \mathcal{J}_{\text{hom}}^\alpha, \quad \mathcal{J}_{\text{hom}}^\alpha := \frac{a_0}{\Omega_0} (R_\beta{}^\alpha \wedge \vartheta^\beta - \Gamma_\beta{}^\alpha \wedge T^\beta), \quad (6)$$

$$d^*\mathcal{F}^\alpha = \Omega_0 \mathcal{J}_{\text{inh}}^\alpha, \quad \mathcal{J}_{\text{inh}}^\alpha := \frac{a_0}{\Omega_0} ({}^*R_\beta{}^\alpha \wedge \vartheta^\beta - \Gamma_\beta{}^\alpha \wedge {}^*T^\beta). \quad (7)$$

In the gravitational sector of Evans' theory, the Einstein-Cartan theory of gravity (EC-theory) was adopted by Evans. Thus, the field equations are those of

Sciama [11, 12] and Kibble [7], which were discovered in 1961:

$$\frac{1}{2} \eta_{\alpha\beta\gamma} \wedge R^{\beta\gamma} = \kappa \Sigma_\alpha = \kappa (\Sigma_\alpha^{\text{mat}} + \Sigma_\alpha^{\text{elmg}}) , \quad (8)$$

$$\frac{1}{2} \eta_{\alpha\beta\gamma} \wedge T^\gamma = \kappa \tau_{\alpha\beta} = \kappa (\tau_{\alpha\beta}^{\text{mat}} + \tau_{\alpha\beta}^{\text{elmg}}) . \quad (9)$$

Here  $\eta_{\alpha\beta\gamma} = \star(\vartheta_\alpha \wedge \vartheta_\beta \wedge \vartheta_\gamma)$ . The total energy-momentum of matter plus electromagnetic field is denoted by  $\Sigma_\alpha$ , the corresponding total spin by  $\tau_{\alpha\beta}$ .

This is the set-up. What we will do here is to propose a new variational principle that describes Evans' theory. We will derive the field equations and will discuss their properties.

## 2 Closing loopholes in Evans' theory

It is apparent that there exist a couple of loopholes in Evans' theory. Apart from announcing the second field equation (9) only verbally and without specifying any formula, the right-hand sides of the two field equations are left open in Evans' approach. How is the energy-momentum  $\Sigma_\alpha^{\text{elmg}}$  of Evans' field defined, how the spin  $\tau_{\alpha\beta}^{\text{elmg}}$ ? Silence is the only answer in Evans' verbose publications. In order to have a better grip on Evans' theory, we decided to develop it a bit further.

From the summary of Evans' theory it becomes clear that the *geometrical* equations (1),(2) and the *gravitational* equations (8),(9) represent the viable EC-theory of gravitation that is distinct from general relativity by an additional spin-spin contact interaction which only acts at very high matter densities, see the review [5]. If the sources in the framework of the EC-theory are the *Maxwell field*  $A$  (with  $F = dA$ ) and some matter fields  $\Psi$ , we have the variational principle

$$\begin{aligned} L_{\text{EC}} = & -\frac{1}{2\kappa} \star(\vartheta_\alpha \wedge \vartheta_\beta) \wedge R^{\alpha\beta} - \frac{1}{2\Omega_0} F_\alpha \wedge \star F^\alpha \\ & + L_{\text{mat}}(\vartheta^\alpha, \Psi^{\alpha\beta\dots}, D\Psi^{\alpha\beta\dots}) . \end{aligned} \quad (10)$$

The matter fields  $\Psi$  are supposed to be minimally coupled to gravity and to electromagnetism. Variation with respect to  $A$  yields the inhomogeneous Maxwell equation  $d\star F = \Omega_0 J$ , with  $J = \delta L_{\text{mat}}/\delta A$ , variation with respect to  $\vartheta^\alpha$  and  $\Gamma^{\alpha\beta}$  the gravitational field equations (8) and (9), with  $\Sigma_\alpha^{\text{elmg}}$  and  $\tau_{\alpha\beta}^{\text{elmg}}$  substituted by  $\Sigma_\alpha^{\text{Maxw}}$  and  $\tau_{\alpha\beta}^{\text{Maxw}}$ , respectively. This is conventional wisdom. Thereby, we also find the canonical energy-momentum and the spin angular momentum of the

Maxwell field:

$$\Sigma_{\alpha}^{\text{Maxw}} := -\frac{\delta L_{\text{Maxw}}}{\delta \vartheta^{\alpha}} = \frac{1}{2\Omega_0} [F \wedge (e_{\alpha}]^* F) - {}^* F \wedge (e_{\alpha}] F)] , \quad (11)$$

$$\tau_{\alpha\beta}^{\text{Maxw}} := -\frac{\delta L_{\text{Maxw}}}{\delta \Gamma^{\alpha\beta}} = 0 . \quad (12)$$

However, in Evans' theory, instead of the Maxwell field, we have Evans' extended electromagnetic field. Then the questions arise what the sources on the right-hand-sides of the gravitational field equations (8) and (9) are and what the extended electromagnetic field may contribute to them. For the time being, we forget Evans' ansatz, that is, we develop a field theoretical model *before Evans' ansatz (3) is substituted*.

## 2.1 Auxiliary Lagrangian

We proceed like in Maxwell's theory. We pick Evans'  $\mathcal{A}^{\alpha}$  potential as the electromagnetic field variable and define the field strength  $\mathcal{F}^{\alpha} = D\mathcal{A}^{\alpha}$ . Then  $D\mathcal{F}^{\alpha} = DD\mathcal{A}^{\alpha} = R_{\beta}^{\alpha} \wedge \mathcal{A}^{\beta}$  is the homogeneous equation (5)<sub>1</sub>. For the inhomogeneous field equation, we propose the auxiliary *Lagrangian* 4-form

$$\mathcal{L} = -\frac{1}{2\Omega_0} (\mathcal{F}_{\alpha} \wedge {}^* \mathcal{F}^{\alpha} + {}^* R^{\alpha\beta} \wedge \mathcal{A}_{\alpha} \wedge \mathcal{A}_{\beta}) . \quad (13)$$

This is the Lagrangian for a massless Lorentz vector valued 1-form field that is non-minimally coupled to the curvature. Variation with respect to  $\mathcal{A}_{\alpha}$  yields

$$D {}^* \mathcal{F}^{\alpha} = {}^* R_{\beta}^{\alpha} \wedge \mathcal{A}^{\beta} , \quad (14)$$

which coincides with (5)<sub>2</sub>. Note that the Lagrangian (13), if Evans' ansatz is substituted, is similar in structure as the improved Evans Lagrangian I, Eq.(77). However, (13) is a pure electromagnetical Lagrangian whereas I, Eq.(77) is purely gravitational.

## 2.2 Energy-momentum

Having recovered the (unsubstituted) electromagnetic field equations, we turn to the energy question. In the EC-theory, we get the energy-momentum by varying the Lagrangian with respect to the coframe:

$$\begin{aligned} \Sigma_{\alpha}^{\text{elmg}} := -\frac{\delta \mathcal{L}}{\delta \vartheta^{\alpha}} &= \frac{1}{2\Omega_0} [\mathcal{F}^{\beta} \wedge (e_{\alpha}]^* \mathcal{F}_{\beta}) - {}^* \mathcal{F}^{\beta} \wedge (e_{\alpha}] \mathcal{F}_{\beta}) \\ &\quad + (\mathcal{A}_{\beta} \wedge \mathcal{A}_{\gamma}) \wedge (e_{\alpha}]^* R^{\beta\gamma}) - {}^* (\mathcal{A}_{\beta} \wedge \mathcal{A}_{\gamma}) \wedge (e_{\alpha}] R^{\beta\gamma})] . \end{aligned} \quad (15)$$

Here we need the master formula of [8] for the commutator of a variation  $\delta$  with the Hodge star operator  $*$ . The energy-momentum is still tracefree as in Maxwell's theory,

$$\vartheta^\alpha \wedge \Sigma_\alpha^{\text{elmg}} = 0, \quad (16)$$

since  $\mathcal{A}^\alpha$  is a massless field, and, perhaps surprisingly, the energy-momentum remains symmetric,

$$\vartheta_{[\alpha} \wedge \Sigma_{\beta]}^{\text{elmg}} = \frac{1}{2\Omega_0} \vartheta_{[\alpha} \wedge [(e_{\beta]}]^\star R^{\gamma\delta}) \wedge (\mathcal{A}_\gamma \wedge \mathcal{A}_\delta) - (e_{\beta]}] R^{\gamma\delta}) \wedge \star(\mathcal{A}_\gamma \wedge \mathcal{A}_\delta)] = 0, \quad (17)$$

as some algebra<sup>1</sup> shows, compare [6], Eqs.(B.5.20) and (E.1.27).

We substitute Evans' ansatz (3) and find

$$\begin{aligned} \Sigma_\alpha^{\text{elmg}} &= \frac{a_0^2}{2\Omega_0} [T^\beta \wedge (e_\alpha]^\star T_\beta) - \star T_\beta \wedge (e_\alpha] T_\beta) \\ &\quad + (\vartheta_\beta \wedge \vartheta_\gamma) \wedge (e_\alpha]^\star R^{\beta\gamma}) - \star(\vartheta_\beta \wedge \vartheta_\gamma) \wedge (e_\alpha] R^{\beta\gamma})]. \end{aligned} \quad (18)$$

By some algebra, the term in the second line can be a bit simplified:

$$\begin{aligned} \Sigma_\alpha^{\text{elmg}} &= \frac{a_0^2}{2\Omega_0} [T^\beta \wedge (e_\alpha]^\star T_\beta) - \star T^\beta \wedge (e_\alpha] T_\beta) \\ &\quad + 2 \star R_{\beta\alpha} \wedge \vartheta^\beta + R^{\beta\gamma} \wedge \eta_{\alpha\beta\gamma}]. \end{aligned} \quad (19)$$

Also after the substitution of Evans' ansatz the energy-momentum remains traceless  $\vartheta^\alpha \wedge \Sigma_\alpha^{\text{elmg}} = 0$  and symmetric  $\vartheta_{[\alpha} \wedge \Sigma_{\beta]}^{\text{elmg}} = 0$ .

## 2.3 Spin angular momentum

Now we turn to spin angular momentum. Since the extended electromagnetic potential  $\mathcal{A}^\alpha$  transforms as a vector under Lorentz transformations, it carries spin, as any other Lorentz vector field. Again we find no help in Evans' work. We vary the Lagrangian (13) with respect to the connection:

$$\tau_{\alpha\beta}^{\text{elmg}} := -\frac{\delta \mathcal{L}}{\delta \Gamma_{\alpha\beta}} = \frac{1}{\Omega_0} \left[ \mathcal{A}_{[\alpha} \wedge \star \mathcal{F}_{\beta]} + \frac{1}{2} D \star (\mathcal{A}_\alpha \wedge \mathcal{A}_\beta) \right]. \quad (20)$$

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<sup>1</sup>Let  $\Phi$  be an arbitrary  $p$ -form in a 4-dimensions RC-space with Lorentzian signature. After applying the formula  $\star(\Phi \wedge \vartheta_\alpha) = e_\alpha]^\star \Phi$  twice, it can be shown that  $\vartheta_{[\alpha} \wedge e_{\beta]}]^\star \Phi = \star(\vartheta_{[\alpha} \wedge e_{\beta]}] \Phi)$ .

If we substitute Evans' ansatz (3), we get

$$\tau_{\alpha\beta}^{\text{elmg}} = \frac{a_0^2}{\Omega_0} \left( \vartheta_{[\alpha} \wedge {}^*T_{\beta]} + \frac{1}{2} \eta_{\alpha\beta\gamma} \wedge T^\gamma \right). \quad (21)$$

Now we apply the exterior covariant derivative to (20):

$$D\tau_{\alpha\beta}^{\text{elmg}} = \frac{1}{\Omega_0} \left[ \underbrace{(\mathcal{F}_{[\alpha} \wedge {}^*\mathcal{F}_{\beta]})}_{=0} - \mathcal{A}_{[\alpha} \wedge D {}^*\mathcal{F}_{\beta]} + \frac{1}{2} DD^*(\mathcal{A}_\alpha \wedge \mathcal{A}_\beta) \right]. \quad (22)$$

After using the inhomogeneous field equation and the Ricci identity, we find

$$D\tau_{\alpha\beta}^{\text{elmg}} = 0. \quad (23)$$

Thus, the spin of the field  $\mathcal{A}^\alpha$ , without contribution of the  $\mathcal{A}^\alpha$ -field's orbital angular momentum, is covariantly conserved. As we see from (17) and (23), angular momentum conservation for the vacuum case is fulfilled:

$$D\tau_{\alpha\beta}^{\text{elmg}} + \vartheta_{[\alpha} \wedge \Sigma_{\beta]}^{\text{elmg}} = 0. \quad (24)$$

### 3 A new variational principle for gravity and extended electromagnetism

Evans' theory is distinguished from the foregoing system by the new ansatz (3) for electromagnetism. Thus, instead of the Maxwell Lagrangian, as in (10), we have to take the new Lagrangian (13) describing the Evans field  $\mathcal{A}^\alpha$ . Adding a Lagrange multiplier piece that enforces Evans' ansatz, we find

$$\begin{aligned} L = & -\frac{1}{2\kappa} {}^*(\vartheta_\alpha \wedge \vartheta_\beta) \wedge R^{\alpha\beta} - \frac{1}{2\Omega_0} (\mathcal{F}_\alpha \wedge {}^*\mathcal{F}^\alpha + {}^*R^{\alpha\beta} \wedge \mathcal{A}_\alpha \wedge \mathcal{A}_\beta) \\ & + L_{\text{mat}}(\vartheta^\alpha, \Psi^{\alpha\beta\dots}, D\Psi^{\alpha\beta\dots}) + \lambda_\alpha \wedge (\mathcal{A}^\alpha - a_0 \vartheta^\alpha). \end{aligned} \quad (25)$$

The Lagrange multiplier is a covector-valued 3-form with 16 independent components. The conserved currents of this model Lagrangian can be derived with the help of the general formalism as developed, e.g., by Obukhov and Rubilar [10].

Let us first discuss the situation when the Lagrange multiplier is put to zero. Then variations with respect to  $\mathcal{A}^\alpha, \vartheta^\alpha, \Gamma^{\alpha\beta}$  lead to the field equations (5)<sub>2</sub>, (8), (9), respectively, that is, apart from Evans' ansatz, we recover the relevant field

equations in electromagnetism and gravitation as they are characteristic for Evans' theory. Insofar the variational principle does what it is supposed to.

Now we relax the multiplier and, accordingly, have a new field variable  $\lambda_\alpha$ . If we *drop the matter fields*, the variation of the Lagrangian (25) looks now as follows:

$$\begin{aligned} \delta L = & \delta \mathcal{A}_\alpha \wedge \left[ -\frac{1}{2\Omega_0} (D^* \mathcal{F}^\alpha - {}^* R_\beta{}^\alpha \wedge \mathcal{A}^\beta) - \lambda^\alpha \right] \\ & + \delta \vartheta^\alpha \wedge \left[ \frac{1}{2\kappa} (-G_\alpha - \kappa \Sigma_\alpha^{\text{elmg}}) + a_0 \lambda_\alpha \right] \\ & + \delta \Gamma^{\alpha\beta} \wedge \left[ \frac{1}{2\kappa} (-C_{\alpha\beta} - \kappa \tau_{\alpha\beta}^{\text{elmg}}) \right] \\ & + \delta \lambda_\alpha \wedge [\mathcal{A}^\alpha - a_0 \vartheta^\alpha] . \end{aligned} \quad (26)$$

Here  $G_\alpha := \frac{1}{2} \eta_{\alpha\beta\gamma} \wedge R^{\beta\gamma}$  is the Einstein 3-form and  $C_{\alpha\beta} := \frac{1}{2} \eta_{\alpha\beta\gamma} \wedge T^\gamma$  the Cartan 3-form, as they were defined in I, Eq.(16) and I, Eq.(15), respectively. They arise also from the variation of the Hilbert type Lagrangian with respect to coframe and connection. The expressions in the brackets have to vanish at the extremum of the action.

The first term yields the value for the multiplier

$$\lambda^\alpha = -\frac{1}{\Omega_0} (D^* \mathcal{F}^\alpha - {}^* R_\beta{}^\alpha \wedge \mathcal{A}^\beta) = -\frac{a_0}{\Omega_0} (D^* T^\alpha - {}^* R_\beta{}^\alpha \wedge \vartheta^\beta) . \quad (27)$$

Consequently, the first field equation of gravitation is modified,

$$G_\alpha = \kappa \Sigma_\alpha^{\text{elmg}} + \frac{a_0^2 \kappa}{\Omega_0} (D^* T_\alpha - {}^* R_{\beta\alpha} \wedge \vartheta^\beta) , \quad (28)$$

whereas the second one remains the same, namely,

$$C_{\alpha\beta} = \kappa \tau_{\alpha\beta}^{\text{elmg}} . \quad (29)$$

We introduce the dimensionless constant<sup>2</sup>

$$\xi := \frac{a_0^2 \kappa}{\Omega_0} , \quad (30)$$

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<sup>2</sup> We determine the dimensions of the different pieces ( $\ell$  dimension of length,  $\mathfrak{h}$  of action,  $q$  of electric charge,  $\phi$  of magnetic flux):

$$[a_0] = \frac{\Phi}{\ell} = \frac{\mathfrak{h}}{q\ell} , \quad [\kappa] = \frac{[\kappa] \mathfrak{h}}{\mathfrak{h}} = \frac{\ell^2}{\mathfrak{h}} , \quad [\Omega_0] = \frac{\mathfrak{h}}{q^2} , \quad \Rightarrow \quad [\xi] = \left( \frac{\mathfrak{h}}{q\ell} \right)^2 \frac{\ell^2 q^2}{\mathfrak{h}} = 1 .$$

which is characteristic for Evans' theory. Using the length  $\ell_E$ , see I, Eq.(25), we have  $a_0 = h/(2e\ell_E)$ . Since  $\kappa = 8\pi G/c^3$ , we find

$$\xi = \frac{h^2}{4e^2\ell_E^2} \frac{8\pi G}{c^3\Omega_0} = 2\pi^2 \frac{2\Omega_{\text{QHE}}}{\Omega_0} \left(\frac{\ell_P}{\ell_E}\right)^2 = \frac{2\pi^2}{\alpha} \left(\frac{\ell_P}{\ell_E}\right)^2 \approx 2705 \left(\frac{\ell_P}{\ell_E}\right)^2, \quad (31)$$

with  $\ell_P := \sqrt{G\hbar/c^3} \approx 10^{-31}m$  as Planck length,  $\Omega_{\text{QHE}} := h/e^2$  as Quantum Hall resistance (von Klitzing constant), see [3], and  $\alpha$  as fine structure constant. Note, since  $G > 0$  and  $\Omega_0 > 0$ , we have always  $\xi > 0$ .

All what is left from Evans' theory, are these 16+24 field equations in which the sources are specified by (19) and (21), but only 10 of the 16 are independent. If we substitute these sources, we find the system

$$-\frac{1}{2}\eta_{\alpha\beta\gamma} \wedge R^{\beta\gamma} = \frac{\xi}{2} \left[ T^\beta \wedge (e_\alpha]^\star T_\beta) - {}^\star T_\beta \wedge (e_\alpha] T_\beta) + R^{\beta\gamma} \wedge \eta_{\alpha\beta\gamma} + 2D {}^\star T_\alpha \right], \quad (32)$$

$$-\frac{1}{2}\eta_{\alpha\beta\gamma} \wedge T^\gamma = \xi \left( \vartheta_{[\alpha} \wedge {}^\star T_{\beta]} + \frac{1}{2}\eta_{\alpha\beta\gamma} \wedge T^\gamma \right). \quad (33)$$

Note that (32) represents partial differential equations of second order in the coframe, because  $D {}^\star T_\alpha = D {}^\star D \vartheta_\alpha$ , and first order in the connection. The linearized version is a wave type equation for the coframe  $\vartheta_\alpha$ .

## 4 Solution of the second field equation

The second (Cartan's) field equation (33) is a homogeneous algebraic equation for the components of the torsion. We can solve this equation exactly. For this purpose we need the identity (for the proof, see [9], for example):

$$\frac{1}{2}\eta_{\alpha\beta\gamma} \wedge T^\gamma \equiv \vartheta_{[\alpha} \wedge h_{\beta]}, \quad (34)$$

$$h_\alpha := {}^\star(-{}^{(1)}T_\alpha + 2{}^{(2)}T_\alpha + \frac{1}{2}{}^{(3)}T_\alpha). \quad (35)$$

The right-hand side of (35) is constructed from the irreducible parts of the torsion. Namely, let us recall that the torsion 2-form can be decomposed into the three irreducible pieces,

$$T^\alpha = {}^{(1)}T^\alpha + {}^{(2)}T^\alpha + {}^{(3)}T^\alpha, \quad (36)$$



where the *vector*, *axial vector* and *pure tensor* parts of the torsion are defined by

$${}^{(2)}T^\alpha = \frac{1}{3}\vartheta^\alpha \wedge (e_\nu \rfloor T^\nu), \quad (37)$$

$${}^{(3)}T^\alpha = -\frac{1}{3}\star(\vartheta^\alpha \wedge \star(T^\nu \wedge \vartheta_\nu)) = \frac{1}{3}e^\alpha \rfloor (T^\nu \wedge \vartheta_\nu), \quad (38)$$

$${}^{(1)}T^\alpha = T^\alpha - {}^{(2)}T^\alpha - {}^{(3)}T^\alpha. \quad (39)$$

Substituting (34) into (33), we find

$$\xi \star T_\alpha + (1 + \xi) h_\alpha = 0. \quad (40)$$

Using then (35) and (36), we can ultimately recast the last equation into the form

$$-{}^{(1)}T^\alpha + (3\xi + 2) {}^{(2)}T^\alpha + \frac{1}{2}(3\xi + 1) {}^{(3)}T^\alpha = 0. \quad (41)$$

The irreducible parts are all algebraically independent. Hence we can conclude that all the three terms in (41) vanish. For generic case of the coupling constant  $\xi$  we thus ultimately find the trivial solution:

$${}^{(1)}T^\alpha = {}^{(2)}T^\alpha = {}^{(3)}T^\alpha = 0, \quad \text{hence} \quad T_\alpha = 0. \quad (42)$$

We may have nontrivial torsion for two exceptional cases. Namely, when

$$\xi = -\frac{2}{3} \quad \text{or} \quad \xi = -\frac{1}{3}. \quad (43)$$

However, since  $\xi > 0$ , these are unphysical cases that can be excluded.

## 5 Conclusions

For the generic case, we substitute the vanishing torsion solution (42) into the first field equation (32) and find that the latter reduces to the usual Einstein equation

$$\frac{1}{2}\eta_{\alpha\beta\gamma} \wedge \tilde{R}^{\beta\gamma} = 0, \quad (44)$$

where the tilde denotes the object constructed from the Riemannian (Christoffel) connection. In this sense, the model under consideration is similar to the Einstein-Cartan theory that also reduces to Einstein's general relativity in absence of the sources with spin, see [5, 13].

This similarity goes even further when the nontrivial matter sources are taken into account. Then the right-hand side of (33) will contain the spin current 3-form  $\tau_{\alpha\beta}^{\text{mat}}$  of the matter fields. Subsequently one can solve the second field equation (33), expressing the torsion in terms of the spin of matter. By substituting this into (32), we can recast the first field equation into a form of the Einstein equation with the *effective* energy-momentum current that will contain the quadratic contributions of spin. The same occurs in the Einstein-Cartan theory, too.

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